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Free Vibration Analysis of Tapered Composite Aircraft Wing via the Finite Element Method

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Abstract

The responses of the structures used in engineering applications under the effects of static and dynamic forces are significant in the design phase. Determination of the response of dynamic forces for a structure is initially performed by the evaluation of free vibration characteristics that are mode shape of the structure and vibration frequencies. This paper presents modal analyses of tapered aircraft wing structures that consist of NACA4415 design and different common materials used in the aviation industry. Furthermore, the effect of winglets on natural frequencies is examined. The main wing structures as ribs and shells are drawn using CATIA and imported to ANSYS Workbench. Analyses have been carried out considering the aircraft wing as a three-dimensional cantilever beam by fixing one end (root chord) of the aircraft wing while the other end (tip chord) is free. The first ten modes of free vibration with their respective natural frequencies and mode shapes of the wing structures of the aircrafts are obtained. The results show that the winglets decrease the natural frequency noticeably and the shell material as Carbon Epoxy UD has been observed to have higher natural frequency compared with Kevlar Epoxy.

Keywords: Free vibration, Natural frequency, Modal analysis, Aircraft wing, Mode shapes

Sonlu Elemanlar Yöntemi ile Kompozit Konik Uçak Kanadının Serbest Titreşim Analizi

Öz

Mühendislik uygulamalarında kullanılan yapıların statik ve dinamik kuvvetlerin etkisi altındaki tepkileri, yapıların tasarım aşamasında önemlidir. Bir yapı için dinamik kuvvetlerin tepkisinin belirlenmesi, öncelikle yapının mod şekli ve titreşim frekansları olan serbest titreşim özelliklerinin değerlendirilmesiyle gerçekleştirilir. Bu makale, havacılık endüstrisinde kullanılan NACA4415 tasarımı ve farklı yaygın malzemelerden oluşan konik uçak kanat yapılarının modal analizlerini sunmaktadır. Ayrıca kıvrık

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kanatçık eklemlerinin (winglets) doğal frekanslar üzerindeki etkisi incelenmiştir. Ana kanat yapıları olan nervür (rib) ve kabuk (shell) yapıları CATIA kullanılarak oluşturulmuştur ve ANSYS Workbench'e aktarılmıştır. Analizler, uçak kanadının bir ucu (kök kiriş) serbestken diğer ucu (uç kirişi) sabitlenerek üç boyutlu bir konsol kiriş olarak düşünülerek yapılmıştır. Uçak kanat yapılarının ilk on doğal frekansları ve ilgili mod şekilleri elde edilmiştir. Sonuçlar; kanat uçlarına kıvrık kanatçık eklenmesinin doğal frekansları oldukça düşürdüğünü ve kabuk malzemesi olarak Karbon Epoksi EY kullanmanın, Kevlar Epoksi kullanımına göre daha yüksek doğal frekanslar oluşturduğunu göstermiştir.

Anahtar Kelimeler: Serbest titreşim, Doğal frekans, Modal analiz, Uçak kanadı, Mod şekilleri

1. INTRODUCTION

The wing of an aircraft has a great influence on the aerodynamic performance of a structure of airplane. Stabilization of an aircraft consists of many aspects such as vibration controls in an airplane wing since dangerous vibrations in different modes such as torsion or stretching may occur. Therefore, it is supreme that the structure of an aircraft wing is designed to avoid the failure.

Modal analysis assists in the determination of the vibration characteristics of mechanical structures. The mechanical analysis of vibration plays a significant role in potential mathematical tools for modeling and problems which may be eliminated in preliminary engineering designs before manufacturing of the systems [1]. Besides vibrational characteristics of a mechanical system consists natural frequencies and mode shapes, the behavior of different parts of the mechanical structure under the conditions of dynamic load may be observed [2]. Furthermore, a description of a mechanical structure is carried out by modal analysis in terms of vibration characteristics which are natural frequency and mode shapes [3].

Modal analysis ensures to determine the reasons for vibrations that may cause damage to the aircraft system and is used in the reduction of the issues. Hence, free vibration analysis of an aircraft wing becomes a compelling problem to be handled during design process. In examination of gust responses, the fundamental aspect is considered to investigate the natural frequencies and modes. The wings of an airplane structure generally consist of non-uniform structures that has swept and tapered design. Non-uniform structure properties of an aircraft wing require advanced modeling due to the effects of complex properties. Thus, free vibration analysis on airplane wing has attracted many scientific research areas to be investigated by.

In the last years, applications of aerospace have increased due to developments in the industry. A new design and developing the existing structural design of an aircraft requires knowledge and technical background. Therefore, much research may be found in the literature that focuses on the wing structure due to being complex assemblies of an aircraft. Erdener and Yaman performed research that aims to have development of an aircraft wing's structural model [4]. A finite element model of a wing was created, and dynamic analyses were performed on the finite element model that concluded the comparison of natural frequencies and mode shapes of the wing with effects on the structure. The aviation industry has taken great development in recent years by attempts to decrease the costs and increasing the efficiency by different research and comparison of proper material selection and lightweight materials were the key such as composites. Composites has many advantages compared to metals and other materials. The main composite materials used in the aircraft industry are reinforced fibers or particles provided by resin matrix [5]. The current presentation of the general wing structure was designed and analyzed by Yang et al. [6]. The study initiates with the sketch of the wing structure and analyses were carried out by using a numerical tool. Later, the aircraft wing optimization was carried out by altered types of material properties and boundary states. Furthermore, the modal analysis was performed for natural frequencies to investigate the vibration characteristics of the wing. The identification of the work was performed by vibration characteristics to investigate an airplane wing. Complex design parameters and hard fabrication restricts experimental projects on aircraft wings. In addition to this, lack of resources for manufacturing results analysis of aircraft wings that are considered as cantilever beams for experimental projects of modal analysis. The investigation of the modal analysis of an aircraft wing was carried out by Khadse and Zaweri [7]. Modeling of the wing structure was handled in ANSYS software and modal analysis was performed by adding a fixed end while the other end was free. Then, the investigation of natural frequency and modes of vibration-assisted comparison for numerical results and analytical frequency of cantilever beam. According to Sureka and Meher [8], the effect of different types of materials for the structure of wings such as ribs, winglets, and shells was investigated. Modeling of a NACA airfoil with ribs was used by using different types of materials and the preference was concluded by aluminum. Banerjee [9] discussed in his research about dynamic behaviors of aircraft wings. Modal analysis of a transport aircraft was investigated and the significance of a deep approach for high aspect ratio wings was indicated by performing modal analysis. The cantilever beam theory was applied on the wing structure by boundary conditions and different mathematical approaches for the wing structure were initially obtained due to the necessity for natural frequencies of the wing. Then, Banerjee concluded the research by discussion and results of the project. Kuntoji and Kuppast [10] presented a study to emphasize the significance of vibration characteristic analysis of an aircraft wing due to being the major focus of a design of an aircraft. A design of an aircraft using NACA standards was discussed, and the modal analysis of the wing was observed in terms of natural frequency for the characteristics of vibration. Due to the principle of aerodynamics, stresses and deformations of a wing affect the structure based on behaviors of vibration characteristics. Hence, understanding the effects of stress on natural frequency had been performed by modal analysis. Saran et al. [11] presented a study for the development of wing structures. Modal analyses were performed for an aircraft wing and natural frequencies were used to avoid resonance on the material for preventing failure. The analysis of natural frequency was obtained by using ANSYS software for calculation resonance which is accurate if it is equal to or more than natural frequency. Therefore, the importance of wing structure was emphasized and simulation boundary conditions with cantilever beam consideration of the wing used in analyses in the research. As stated by Günay and Özbay [12], a wing of an aircraft is the responsible component for lift generation. A wing overcomes the weight lift due to the gravity with the lift force that it produces and keeps the aircraft in the air. Therefore, lifting force is created by the aerodynamic profile of the wing. In their research, static analysis performance was handled to determine stress components by applications of load and modal analysis of the wing was performed for the determination of natural frequencies on the wings and oriented deformations owing to the frequencies. Furthermore, a study on an airplane wing based on the concept of a consideration as a cantilever beam was performed by Demirtaş and Bayraktar [13].

The finite element method has been used to find approximate results of boundary value problems and is one of the most common computational techniques for the wide variety of problems in different engineering areas such as mechanical and aerospace [14,15], structural [16,17], geological [18], civil engineering [19,20] and so on.

As the seen from the literature, the previous studies the related to free vibration of airplane wings have simplified the wing model due to some restrictions as the computational burden of the analysis, complexity of the real-life wing geometry and the complex analytical and/or numerical mathematical manipulations. In this work, a life-size wing of an airplane is properly modeled with inside and outside structures. The wing as well as the inside ribs are tapered through the thickness and NACA 4415 profile is used to construct the airfoil. Also, the different winglet designs are considered. The natural frequencies and mode shapes related to the wing structure have been obtained using finite element software package and the effects of different materials and winglet designs on the free vibration of wing are discussed.

The results may be used to benchmark the other studies and future works in this field is the main aim.

2. DESIGN OF WING

Determination of wing profile which depends on the type and parameters has an incontrovertible effect on the execution of aircraft production. Therefore, the most crucial part of an aircraft is the wing structure. The aircraft wing has modeled using CATIA and ANSYS Design Modeler where the airfoil coordinates are imported from NACA profile. The model of the wing is a tapered model wing and has inner structures as ribs.

In this work, NACA 4415 profile is selected. Here, maximum camber is expressed as the chord's percentage in the first digit. Second digit denotes the position of maximum camber according to the chord and the last two digits represent the thickness of the selection of airfoil. Therefore, the selected airfoil type which has a maximum thickness of 15% at 30.9% chord and maximum camber 4% at 40.2% chord depicted in Figure 1.

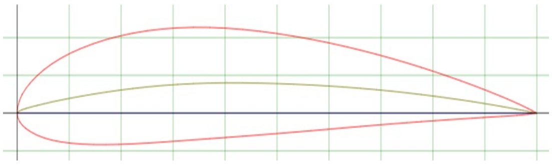


Figure 1. The plot of a NACA 4415 foil

The wing-type used for the research purposes has been inspired by the wing of a one of commercial aircrafts. On the other hand, some parameters are altered for research purposes. The altered dimensions of the aircraft wing used are given in Table 1.

Table 1. Wing design parameters

Parameters	Dimension (m)
Wingspan	7.0
Root length	0.713
Tip length	0.432

The research focuses on modal analyses and comparison of the results for 3 different types of wing design that are divided into four sections. The material selection and dimensions of the wings for each case have remained the same while cases differ in three different types of wings (without winglet, single winglet, and double winglet). The thicknesses of wing structures are taken as 3, 4 and 5 mm for winglet, rib, and shell, respectively.

Importation of data into ANSYS Workbench [21] is followed by the data and dimensions of the coordinates of the airfoil curvation and generation. The isometric view of the wing is shown in Figure 2.

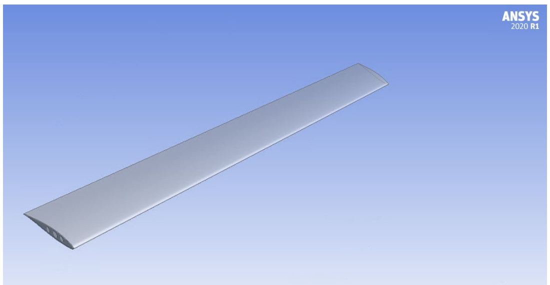


Figure 2. Aircraft wing CAD model in ANSYS Workbench

Aircraft wing material selection for the research has been carried out by extensive investigation of common materials in aviation sector. For research purposes, Kevlar-Epoxy and Carbon-Epoxy composite (Epoxy Carbon UD 395 GPa) from the software library of ANSYS Workbench are used for shell structures. Besides, aluminum (Al) and Titanium alloy are selected as winglet and rib materials.

Accordingly, four different cases have been assigned by altering the material type of the wing structures. The material selection for the wings and the properties of the materials are represented in Table 2.

Table 2. Material properties of the materials

Material properties	**Materials**			
	Epoxy carbon UD (395 GPa)	Kevlar epoxy	Aluminum (Al)	Titanium alloy
E_1 (GPa)	209	75.8	70	96
E_2 (GPa)	9.45	5.51	70	96
ν_{12}	0.27	0.30	0.33	0.36
ρ(kg/m^3)	1600	1330	2707	4620

The modal analyses have been performed for four different categorized cases for three different wing designs that are without winglet, single winglet, and double winglet. Therefore, each case consists of three different aircraft wings with altered material properties and cases are numbered as 1,2,3, and 4. Each case and detailed representation are shown in Table 3 where each case number consists of the different wing designs of without winglet, single winglet, and double winglet.

Table 3. The materials of the wing structures for each case

	Case Numbers			
Wing structures	**Case 1**	**Case 2**	**Case 3**	**Case 4**
Shell	Epoxy carbon UD (395 GPa)	Epoxy kevlar	Epoxy carbon UD (395 GPa)	Epoxy kevlar
Rib	Ti	Ti	Al	Al
Winglet	Al	Al	Al	Al

3. ANALYSIS OF THE WING

One of the basic concepts of finite-element (FE) is a body structure of a system that may be divided elements of finite dimension. The main idea of FE method is defined by continuous quantities namely temperature, pressure, or displacement. Hence, any continuous quantity approximation may be approached as a discrete model that is composed of continuous set functions. In addition to this, the functions in series approach the precise solution [14].

The design of the aircraft wing consists of ribs, upper-lower skins (shells), and winglets. The internal and external structures of the wing had been modeled by different element type selections. In the modeling of the upper-lower skins and winglets, the Shell281 element was used. Shell281 is a highly recommended element type in the designs of aircraft wings since it is suitable for modeling of thin and axisymmetric structures [22]. This element type has translation and rotational movements in the x, y, and z directions. Therefore, the element has eight nodes with six degrees of freedom (DOFs) at each node [23]. Furthermore, rib structures are modeled by solid element selection. The 3D element type was applied since solid bodies and structures are provided by their minimum simplifications depending on the geometry. Selected Solid185 element type has eight nodes having three degrees of freedom at each node: translations in the nodal x, y, and z directions with high strain and large deflection capabilities [23].

The aircraft design includes in 2D (shell) and 3D (solid) structures. Therefore, the interface between structures which has different degrees of freedom requires an additional step during analysis. The translational DOFs were transferred (shared) within 2 element types on the contact points and rotations were not transferred across there though since solid elements do not consist rotational DOF. Hence, the shared nodes between solid and shell structures were combined by the node merge. The mesh parameters are shown in Table 4 while the mesh models of the wings are shown in Figure 3 and Figure 4. As the example of number of node and element generated in the study, the maximum values of node and element number obtained in Case 1 double winglet analysis are given here as 128206 and 47240, respectively.

Table 4. A sample of mesh details of the wing model for Case 1 double winglet

Mesh Parameters and Mesh Methods	
Span angle center	Fine
Shell mesh	Face + Tetra meshing
Rib mesh	Hexa meshing
Winglet mesh	Tetra meshing
Mesh size	25 mm
Smoothing	Fine
Number of nodes – 128296	
Number of elements – 47240	

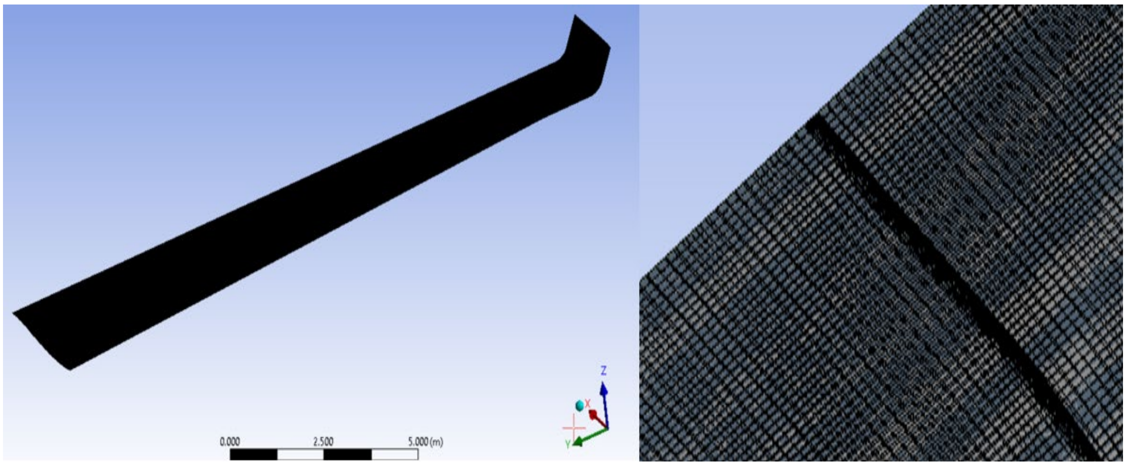


Figure 3. Generated mesh of single-winglet wing design

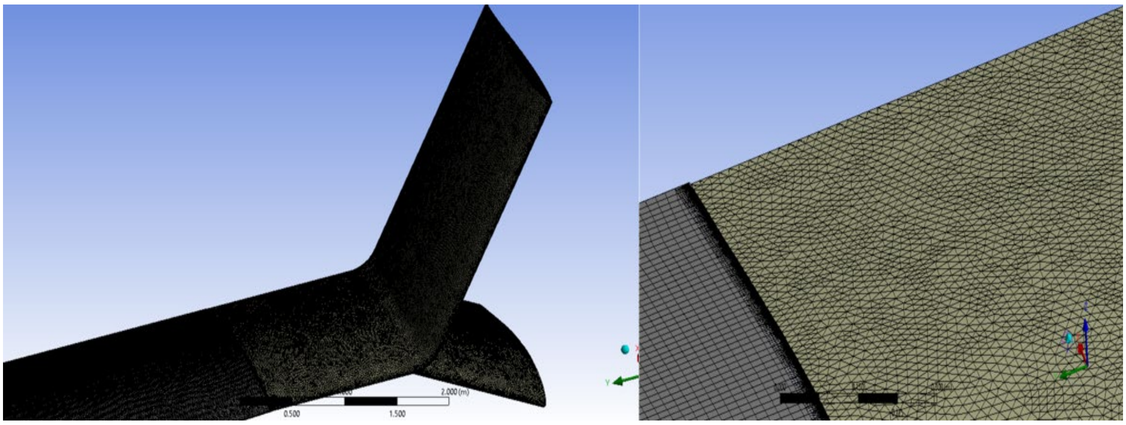


Figure 4. Generated mesh of double-winglet wing design

Due to the lack of manufacturing resources and cost of high fabrication of an aircraft wing, validation of modal analysis for a wing is compelling. Therefore, consideration of an aircraft wing has been common in literature since the chord at the root is fixed into the fuselage within the assistance of spars while the tip chord is free. In this study, the wing models are set up at the conditions of the cantilever beam and the condition has been based on an objective that the wing root is attached to the fuselage body and therefore results as fixed. The fixed support is applied to the wing root face.

4. RESULTS AND DISCUSSION

Modal analyses are performed and the number of modes to be obtained has been set to 10 to observe the ten-mode shapes of the given wing structures under the cantilever beam condition. The total deformation obtained in all modes of the aircraft wing enabled us to reach the vibration behavior of the wing structure. Based on the results and mode shapes, bending modes and torsional modes are observed and interpreted. The analyses have been performed for structures without winglet, single winglet, and double winglet. The first 10 natural frequencies given in Table 5-8 and their respective mode shapes are obtained for each case, however, in order to prevent the redundancy, first three fundamental mode shapes for Case 1 are illustrated only. These mode shapes are depicted in Figure 5-7. The first three bending mode shapes are clearly seen from Figure 5 for the without winglet design. In the case of the winglet addition, the modes are again bending modes, however, the intensity of vibration occurs at the winglets for both single and double cases. This behavior becomes obvious especially at the third mode shapes in Figure 6 and 7 and explains the big differences between the natural frequency values of third mode of with and without winglet cases given in Table 5. Note that the same mode numbers for different wing-tips in Tables 5-8 may not indicate the same vibration mode such as bending or torsion. In order to discuss the variation of natural frequency values of same vibration mode of same cases with the different wing-tip design which is not the main focus of this study, the bending modes, torsional modes and coupled modes should be defined and tabulated separately by using mode shapes first.

It can be seen from the observation and comparison of Table 5 with 6 and Table 7 with 8, the shell material Carbon Epoxy UD has been obtained to have higher natural frequency compared to Kevlar Epoxy. Also, the decrease on the natural frequency is observed by using single winglet and the decrease becomes more considerably with the double winglet design. Additionally, as the mode number increases error in the computation of natural frequency increases. This may have been caused by the meshing quality that affects greatly a FEA results.

Table 5. Natural frequencies of the wing models for Case 1

Mode	**Natural frequency (Hz)**		
	Without winglet	**Single winglet**	**Double winglet**
1	5.5034	4.5318	2.422
2	25.198	17.327	13.548
3	82.211	31.214	29.338
4	119.75	70.404	55.915
5	144.52	108.94	82.846
6	200.8	142.07	128.78
7	232.84	204.56	188.13
8	270.08	261.53	210.73
9	291.33	274.02	293.32
10	374.3	298.65	304.25

Table 6. Natural frequencies of the wing models for Case 2

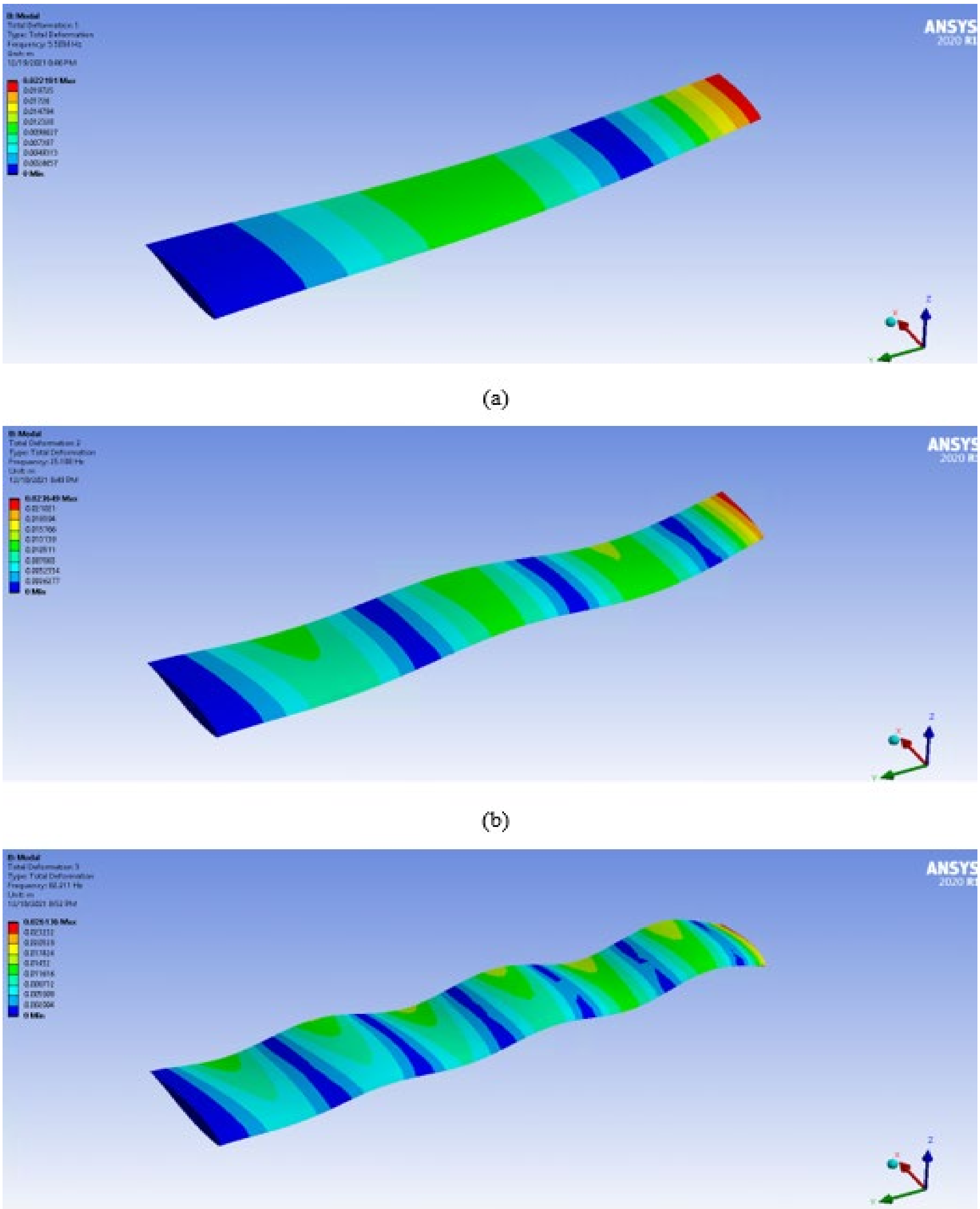
Mode	**Natural frequency (Hz)**		
	Without winglet	**Single winglet**	**Double winglet**
1	3.3903	7.1669	4.137
2	20.966	22.44	16.821
3	35.397	32.06	42.537
4	84.963	62.16	70.492
5	106.29	112.57	131.01
6	155.43	144.54	163.04
7	194.17	199.42	195.45
8	211.06	207.69	212.91
9	240.07	228.52	247.14
10	272.89	251.5	295.22

Table 7. Natural frequencies of the wing models for Case 3

Mode	**Natural frequency (Hz)**		
	Without winglet	**Single winglet**	**Double winglet**
1	7.8717	5.4326	3.4212
2	34.46	19.717	8.4885
3	60.06	46.449	29.337
4	108.02	74.894	55.93
5	166.86	122.15	104.71
6	199.27	162.99	151.74
7	201.1	200.96	198.27
8	233.09	220.42	220.47
9	288.72	260.46	273.68
10	342.19	314.81	297.91

Table 8. Natural frequencies of the wing models for Case 4

Mode	**Natural frequency (Hz)**		
	Without winglet	**Single winglet**	**Double winglet**
1	3.3935	3.5479	2.302
2	14.429	10.254	11.821
3	35.455	29.969	29.148
4	66.319	62.236	64.604
5	91.203	82.573	103.57
6	155.63	125.6	134.98
7	185.77	153.5	184.51
8	211.4	208.72	200.2
9	240.55	225.91	230.62
10	292.95	305.08	273.92



(a)

(b)

(c)

Figure 5. The first three mode shapes of Case 1 without winglet. (a) Mode 1. (b) Mode 2. (c) Mode 3

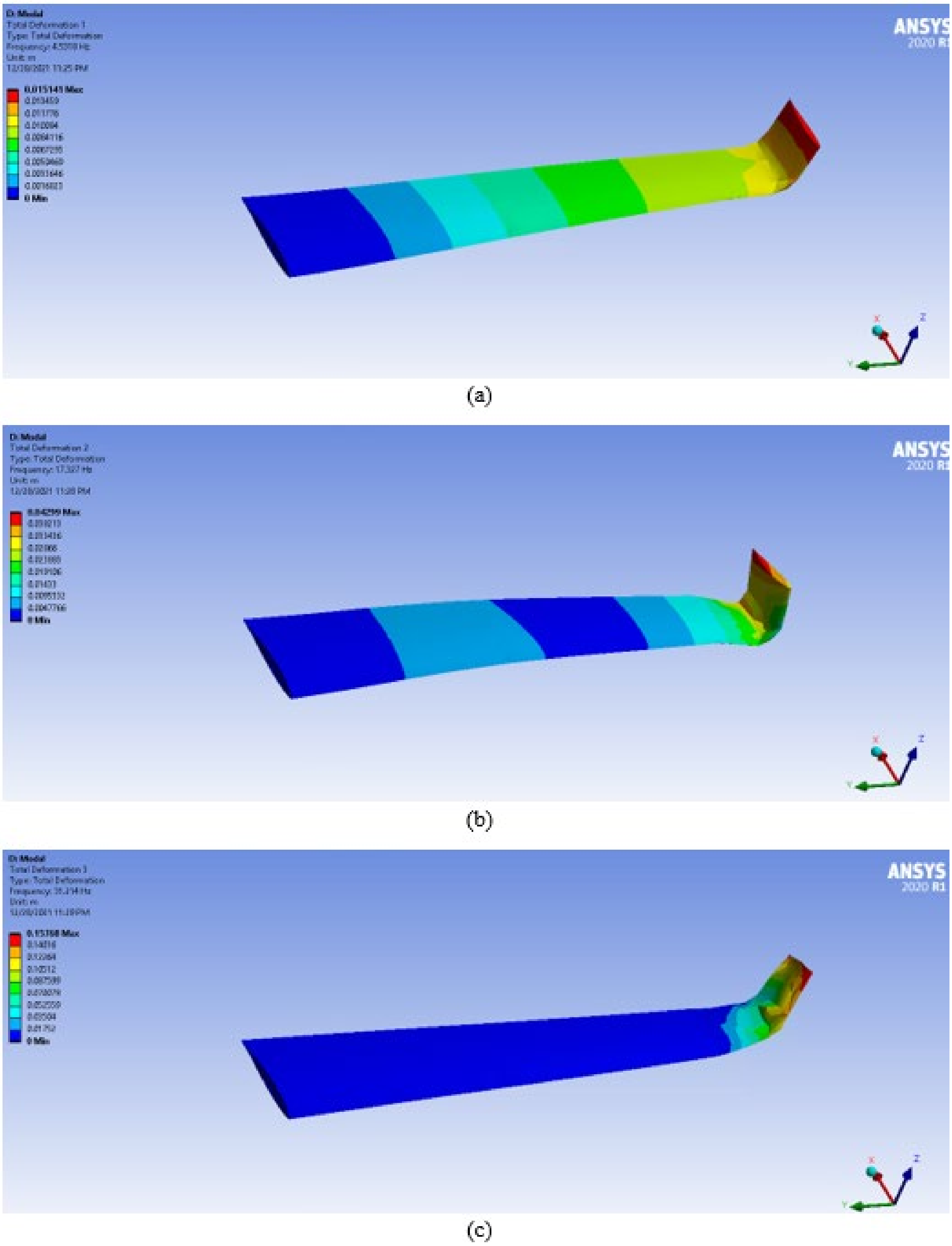
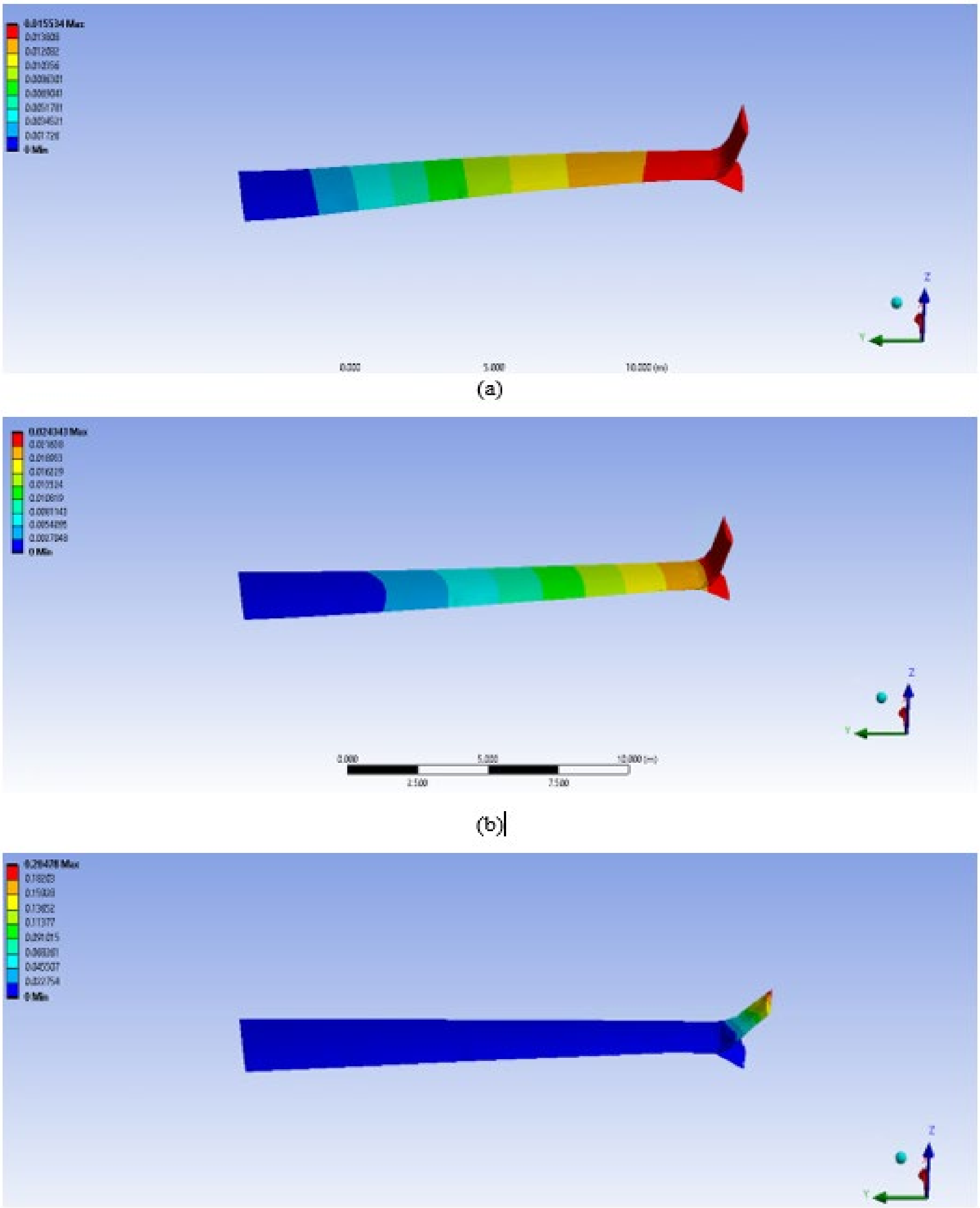


Figure 6. The first three mode shapes of Case 1 with single winglet. (a) Mode 1. (b) Mode 2. (c) Mode 3



(a)

(b)

(c)

Figure 7. The first three mode shapes of Case 1 with double winglet. (a) Mode 1. (b) Mode 2. (c) Mode 3

5. CONCLUSION

In this study, the aircraft wing has been modeled using CATIA and ANSYS Design Modeler and is imported into ANSYS Workbench. A real-life wing design with its inside ribs, outside shell and tapered geometry is considered. The natural frequency of free vibration has been studied using the modal analysis tool. The effect of wingtips on a wing structure of an airplane is also investigated by different cases and material selections.

The aircraft wing simulation has been performed on a non-winglet aircraft wing, single winglet, and double winglet designed wings. For each case, various common materials in aviation sector are defined on inside and outside parts of the wings. Then, simulation is performed and results of the three wing types concerning their specific materials have been compared. According to the obtained results, the natural frequencies of the wings with winglets are obtained to be less than without winglet wing structure results. The results presented in this paper may be used to benchmark future studies in this field.

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